

Experimentally, we find $\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$, which has magnitude $B = \frac{\mu_0 |q|v \sin \phi}{4\pi r^2}$.

\vec{B} is the magnetic field (in T) *caused* by a moving point charge.

$\mu_0 \equiv 4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}$ exactly.

q is its charge (in C, where $1 \text{ C} \equiv 1 \text{ A}\cdot\text{s}$) and \vec{v} is its constant velocity (in $\frac{\text{m}}{\text{s}}$).

r is the distance (in m) from the source point (the point charge) to the field point.

\hat{r} is a unit vector directed *from* the source point *to* the field point. A unit vector has no unit but has magnitude one (unity).

ϕ is the angle between the directions of \vec{v} and \hat{r} (as in Fig. 28.1a).

Cover up the solution and carefully work Example 28.1.

The **law of Biot and Savart**, $d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$, has magnitude $dB = \frac{\mu_0 I dl \sin \phi}{4\pi r^2}$. Of course, to

find \vec{B} , we perform the vector integral of $d\vec{B}$: $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$.

$d\vec{B}$ is the infinitesimal magnetic field (in T) *caused* by a current I (in A) flowing in an infinitesimal length $d\vec{l}$ (in m) (as illustrated with an exaggerated length dl in Fig. 28.3a).

r is the distance (in m) from the source point (the infinitesimal length dl) to the field point and \hat{r} is the corresponding unit vector.

ϕ is the angle between the directions of $d\vec{l}$ and \hat{r} .

These preceding vector equations tell us that the magnetic field is zero directly ahead of ($\phi = 0$) or directly behind ($\phi = 180^\circ$) a moving point charge or a bit of current.

Cover up the solution and carefully work Example 28.2.

Outside a long straight wire, the law of Biot and Savart gives $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance from the *center* of the wire to the field point. A long straight wire's magnetic field lines are circles centered on the wire. **Mentally grasp the wire with your right hand, with your extended thumb in the direction the current flows. Your fingers then wrap around in the directions of \vec{B} .** (See Fig. 28.6.)

Cover up the solutions and carefully work Examples 28.3 and 28.4

From Fig. 28.9, **parallel currents attract, but antiparallel (opposite) currents repel.**

On the axis of a flat circular coil of N turns, the law of Biot and Savart gives $B = \frac{\mu_0 NI a^2}{2(x^2 + a^2)^{3/2}}$, where x

is the distance (in m) along the axis from the center of the coil to the field point and a is the coil's radius (in m). At

the coil's center (that is, at $x = 0$), this equation reduces to $B = \frac{\mu_0 NI}{2a}$.

Cover up the solution and carefully work Example 28.6.

The direction of \vec{B} on the axis of a circular coil is the same direction as the coil's magnetic dipole moment and area vectors: **Wrap the fingers of your right hand around the coil the way the current flows. Then your extended right thumb points along the coil's axis in the direction of \vec{B} .**

Ampere's law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$, where I_{encl} is the net *constant* current (in A) enclosed by the path of the integral. See how we use Ampere's law in the high symmetry Examples 28.7, 28.8, 28.9, and 28.10.

A **solenoid** is a helical coil wrapped on a cylinder. When its length is much greater than its diameter, Example 28.9 shows, near its center, $B = \mu_0 n I$, where n is the number of turns per length (in m^{-1}).

A **toroidal solenoid** (more commonly called a **toroid**) is a coil wrapped on a doughnut-shaped core. Example 28.10 shows $B = \frac{\mu_0 N I}{2\pi r}$, where N is the number of turns (no unit) and r is the distance (in m) from the center to the field point (see Fig. 28.25). This B is the magnitude of the tangential magnetic field (in T) in the "dough" of the doughnut-shaped core (that is, within the turns).

All our previous equations containing μ_0 assume any materials present to be essentially nonmagnetic.

The magnetic dipole moments of atoms are caused *mainly* by the orbital and spin motions of their electrons (nuclear magnetism is about 10^3 times smaller).

The **magnetization** \vec{M} of a material is its net magnetic dipole moment per volume.

Outside of a magnet, its own magnetic field is away from its N-pole and toward its S-pole. In general, this magnetic field decreases with distance from the magnet.

Paramagnetism is the temperature-dependent lining up of the atomic magnetic dipoles when placed in an external magnetic field. In the material, except at very low temperatures, paramagnetism gives only a slight increase over the external magnetic field value.

Diamagnetism is an induced effect that ordinarily gives a weak magnetization that opposes and slightly decreases the value of the external magnetic field in the material. It can be a strong effect in superconductors.

In **ferromagnetism**, adjacent atomic magnetic dipoles line up in strong parallelism in regions called **magnetic domains**. In unmagnetized ferromagnetic material, those domains have random orientations. An external magnetic field causes those domains to grow and/or rotate to give a large magnetization. Figure 28.28 shows a **magnetization curve** for a ferromagnetic material. On the graph, B_0 is the component of the magnetic field that we'd have if no material were present and M is the component of the magnetization \vec{M} in \vec{B}_0 's initial direction. When $M = M_{\text{sat}}$ (where sat is short for *saturation*), the domains are as aligned as possible.

If the domains tend to remain aligned even after the external magnetic field is removed, we have the phenomenon called *hysteresis*, which gives us permanent magnets and magnetic memory materials. Figure 28.29 shows a different **hysteresis loop** for each of three applications.