PHYS 242 BLOCK 8 NOTES

Sections 15.1 to 15.7

A **mechanical wave** travels through some medium (a gas, a liquid, a solid, ...). As the wave passes by, the particles of the medium are displaced away from equilibrium and back ... . (See www.ncat.edu/~gpii.)

In a **transverse wave**, the particles oscillate perpendicular (transverse) to the direction the wave moves.

In a **longitudinal wave**, the particles oscillate along the direction the wave moves.

As Fig. 15.4 shows, a periodic wave travels one wavelength λ in one period of time T, so its wave speed is $\upsilon = \frac{\lambda}{T}$. Remember from simple harmonic motion that $\boxed{f = \frac{1}{T}}$, so $\boxed{\upsilon = \lambda f}$ for *all* periodic waves. The scalar quantities υ, λ, f, and T are never negative.

υ is the **wave speed** (in $\frac{\text{m}}{\text{s}}$).

λ is the **wavelength** (in m)—the spatial length of one complete wave.

f is the frequency (in Hz = hertz) 1 Hz = 1 (complete wave or oscillation or cycle)/s = 1 s^{-1}.

T is the period (in s)—the time for one complete wave or oscillation or cycle.

Cover up the solution and carefully work Example 15.1.

Recall that $\boxed{\omega \equiv 2\pi f}$, where ω is the **angular frequency** (in $\frac{\text{rad}}{\text{s}}$).

Also, we define $\boxed{k \equiv \frac{2\pi}{\lambda}}$, where k is the **wave number** (in $\frac{\text{rad}}{\text{m}}$).

The scalar quantities ω and k are also never negative. The "2π" in these two equations is 2π radians.

Now let's find the wave speed υ in terms of ω and k: $\upsilon = \lambda f = \frac{2\pi}{k}\frac{\omega}{2\pi}$, so $\boxed{\upsilon = \frac{\omega}{k}}$.

The term *sinusoidal* includes cosines as well as sines. We now consider sinusoidal functions of x and t, $y(x,t)$, for waves. At the origin (where $x = 0$), let $y(0,t) = A \cos \omega t$. For waves moving in the $+x$-direction, at some position x (in m) and time t (in s), we have a displacement component $y(x,t)$ that occurred at the origin at the *earlier* time $t - \frac{x}{\upsilon}$, giving $y(x,t) = A \cos \omega(t - \frac{x}{\upsilon}) = A \cos \omega(\frac{x}{\upsilon} - t) = A \cos (\frac{\omega x}{\omega/k} - \omega t) = A \cos (kx - \omega t)$. For waves moving in the $-x$-direction, we simply replace the minus in the parentheses with a plus. That is,

$\boxed{y(x,t) = A \cos (kx - \omega t)}$ is the **wave function** for a sinusoidal wave moving in the $+x$-direction and

$\boxed{y(x,t) = A \cos (kx + \omega t)}$ is the **wave function** for a sinusoidal wave moving in the $-x$-direction.

A is the **amplitude** (in m)—the maximum displacement component (y_{max}) from equilibrium.

$(kx \mp \omega t)$ (usually in radians) is the **phase** of the motion.

Cover up the solution and carefully work Example 15.2 using Eq. (15.7) rather than Eq. (15.4).

For a particle of the medium:

1. Its displacement component from equilibrium is $y(x,t) = A \cos (kx \mp \omega t)$.
2. Its velocity component is $\upsilon_y = \frac{\partial y}{\partial t} = \pm\omega A \sin (kx \mp \omega t)$, so $(\upsilon_y)_{\text{max}} = \omega A$.
3. Its acceleration component is $a_y = \frac{\partial \upsilon_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A \cos (kx \mp \omega t) = -\omega^2 y$, so $(a_y)_{\text{max}} = \omega^2 A$.

Equation 15.12 is called the **wave equation**.

If we move along with a wave crest, we'll see the string (or wire or cable or rope or spring or …) moving past us at the wave speed υ. We can accurately fit a *very* small length L of the crest to a circle of radius R. The weight of this length is negligible, so $\Sigma F_y = ma_y$ becomes $-2F \sin\theta = m\left(-\frac{\upsilon^2}{R}\right)$. Since θ is *very* small, we can accurately replace $\sin\theta$ with θ (in rad), where $\theta = \frac{L/2}{R}$. Also, $m = \mu L$. Substituting into $-2F \sin\theta = m\left(-\frac{\upsilon^2}{R}\right)$ solves to $\boxed{\upsilon = \sqrt{\frac{F}{\mu}}}$. The quantities υ, F, and μ are never negative.

υ is the wave speed (in $\frac{\text{m}}{\text{s}}$) on the string (or wire or cable or rope or spring or …).

F is the magnitude of the tension (in N), which is the force stretching the string … .

μ is the mass per length, $\frac{m}{L}$ (in $\frac{\text{kg}}{\text{m}}$, NOT $\frac{\text{g}}{\text{m}}$) of the string … . Note that L is NOT the self-inductance. Also note μ is NOT the metric prefix 10^{-6}, NOT the magnetic dipole moment NIA, and NOT the permeability.

Cover up the solutions and carefully work Example 15.3.

Section 15.5 derives the following for sinusoidal waves on a string … : $\boxed{P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2}$.

P_{av} is the average power (in W = watt) carried by a transverse sinusoidal wave moving along a string … .

Cover up the solution and carefully work Example 15.4.

A **traveling wave** is simply one that travels from one place to another. As illustrated in Fig. 15.19a, when a traveling wave on a string … comes to a fixed end, it is inverted (π rad phase change) upon reflection. However, if it comes to a free end, as in Fig. 15.19b, it is *not* inverted (zero phase change) upon reflection.

For linear materials, the **principle of superposition** holds: When two or more waves overlap, the resulting displacement is the vector sum of the displacements of the individual waves.

Interference is the result of two or more waves adding together at a point. In **constructive interference**, the resulting amplitude is greater. In **destructive interference**, the resulting amplitude is smaller. A **node** is a point of zero motion (no displacement). An **antinode** is a point of maximum magnitudes of motion and displacement.

A **standing wave** is one that has no net wave velocity and transmits no net power. A standing wave is made up of two traveling waves that are identical except for opposite directions and a possible phase difference. For example, $y_1(x,t) = A\cos(kx - \omega t)$ moving in the $+x$-direction, then reflecting from a fixed end (and inverting) to become $y_2(x,t) = -A\cos(kx + \omega t)$. By the principle of superposition, $y(x,t) = y = y_1(x,t) + y_2(x,t)$. Using the trig identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$, we have the wave function of a standing wave on a string … $\boxed{y = (2A\sin kx)\sin\omega t}$. Here A is the amplitude (in m) of *either* traveling wave and $2A$ is the amplitude (in m) of the standing wave at its antinode(s).

As illustrated in Fig. 15.23e, for a standing wave: 1. the nodes and antinodes alternate, 2. the distance from a node to its nearest antinode is one-quarter wavelength ($\frac{\lambda}{4}$) and, 3. the distance from a node to its nearest node (or from an antinode to its nearest antinode) (if they exist) is one-half wavelength ($\frac{\lambda}{2}$).

Cover up the solution and carefully work Example 15.6 (where $A_{\text{SW}} = 2A$).